Topic 1

Units, Trigonometry, and Vectors

QUICK QUIZZES

1.1 Choice (c). The largest possible magnitude of the resultant occurs when the two vectors are in the same direction. In this case, the magnitude of the resultant is the sum of the magnitudes of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$: $R = A + B = 20$ units. The smallest possible magnitude of the resultant occurs when the two vectors are in opposite directions, and the magnitude is the difference of the magnitudes of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$: $R = | A - B | = 4$ units.

1.2

Vector	x-component	y-component
$\vec{\mathbf{A}}$	−	+
$\vec{\mathbf{B}}$	+	−
$\vec{\mathbf{A}}+\vec{\mathbf{B}}$	−	−

1.3 Vector $\vec{\mathbf{B}}$. The range of the inverse tangent function includes only the first and fourth quadrants (i.e., angles in the range $-\pi/2 < \theta < \pi/2$). Only vector $\vec{\mathbf{B}}$ has an orientation in this range.

ANSWERS TO EVEN NUMBERED CONCEPTUAL QUESTIONS

1.2 Atomic clocks are based on the electromagnetic waves that atoms emit. Also, pulsars are highly regular astronomical clocks.

1.4 **(a)** ~0.5 lb ≈ 0.25 kg or ~10^{-1} kg

(b) ~4 lb ≈ 0.25 kg or ~10^{0} kg

(c) ~4 000 lb ≈ 2 000 kg or ~10^{3} kg

1.6 Let us assume the atoms are solid spheres of diameter 10^{-10} m. Then, the volume of each atom is of the order of 10^{-30} m^3. (More precisely, volume = $4\pi r^3/3$.) Therefore, since 1 cm^3 = 10^{-6} m^3, the number of atoms in the 1 cm^3 solid is on the order of $10^{-6}/10^{-30} = 10^{24}$ atoms. A more precise calculation would require knowledge of the density of the solid and the mass of each atom. However, our estimate agrees with the more precise calculation to within a factor of 10.

1.8 Choice (d). For an angle θ from 0° to 360°, the sine and cosine functions take the values

$$-1 \le \sin\theta \le 1 \quad \text{and} \quad -1 \le \cos\theta \le 1$$

1.10 In the metric system, units differ by powers of ten, so it's very easy and accurate to convert from one unit to another.

1.12 Both answers (d) and (e) could be physically meaningful. Answers (a), (b), and (c) must be meaningless since quantities can be added or subtracted only if they have the same dimensions.

1.14 The correct answer is (a). The second measurement is more precise but, given the number of reported significant figures, each measurement is consistent with the other.

1.16 The components of a vector will be equal in magnitude if the vector lies at a 45° angle with the two axes along which the components lie.

ANSWERS TO EVEN NUMBERED PROBLEMS

1.2 **(a)** L / T^2 **(b)** L

1.4 All three equations are dimensionally incorrect.

1.6 **(a)** $kg \cdot m/s$ **(b)** $Ft = p$

1.8 58

1.10 **(a)** 22.6 **(b)** 22.7 **(c)** 22.6 is more reliable

1.12 **(a)** 3.00×10^8 m/s **(b)** $2.997\,9 \times 10^8$ m/s

(c) $2.997\,925 \times 10^8$ m/s

1.14 **(a)** $346 \text{ m}^2 \pm 13 \text{ m}^3$ **(b)** $66.0 \text{ m} \pm 1.3 \text{ m}$

1.16 **(a)** 797 **(b)** 1.1 **(c)** 17.66

1.18 132 m^2

1.20 3.09 cm/s

1.22 **(a)** 5.60×10^2 km $= 5.60 \times 10^5$ m $= 5.60 \times 10^7$ cm

(b) 0.491 2 km $= 4.912 \times 10^4$ cm

(c) 6.192 km $= 6.192 \times 10^3$ m $= 6.192 \times 10^5$ cm

(d) 2.499 km $= 2.499 \times 10^3$ m $= 2.499 \times 10^5$ cm

1.24 10.6 km/L

1.26 9.2 nm/s

1.28 2.9×10^2 m^3 $= 2.9 \times 10^8$ cm^3

1.30 2.57×10^6 m^3

1.32 ~10^8 steps

1.34 ~10^8 people with colds on any given day

1.36 **(a)** 4.2×10^{-18} m^3 **(b)** ~10^{-1} m^3 **(c)** ~10^{16} cells

1.38 10^{-14} kg

1.40 2.2 m

1.42 8.1 cm

1.44 $\Delta s = \sqrt{r_1^2 + r_2^2 - 2r_1 r_2 \cos(\theta_1 - \theta_2)}$

1.46 2.33 m

1.48 **(a)** 1.50 m **(b)** 2.60 m

1.50 8.60 m

1.52 1.44×10^3 m

1.54 **(a)** 6.1 units at $\theta = +113°$ **(b)** 15 units at $\theta = +23°$

1.56 **(a)** 484 km **(b)** 18.1° N of W

(c) Because of Earth's curvature, the plane does not follow straight lines.

1.58 $\vec{\mathbf{R}} = 9.5$ units at 57° above the $+x$-axis

1.60 **(a)** 13.4 m **(b)** 19.9 m

1.62 1.31 km northward, 2.81 km eastward

1.64 **(a)** 10.0 m **(b)** 15.7 m **(c)** 0

1.66 42.7 yards

1.68 788 mi at 48.0° N of E

1.70 **(a)** 185 N at 77.8° from the x-axis **(b)** 185 N at 258° from the x-axis

1.72 **(a)** 1.609 km/h **(b)** 88 km/h **(c)** 16 km/h

1.74 **(a)** 7.14×10^{-2} gal/s **(b)** 2.70×10^{-4} m^3/s

(c) 1.03 h

1.76 **(a)** $A_2/A_1 = 4$ **(b)** $V_2/V_1 = 8$

1.78 **(a)** 600 yr **(b)** 7×10^4 times

1.80 1×10^6

PROBLEM SOLUTIONS

1.1 Substituting dimensions into the given equation $T = 2\pi\sqrt{\ell / g}$, and recognizing that 2π is a dimensionless constant, we have

$$[T] = \sqrt{\frac{[\ell]}{[g]}} \quad \text{or} \quad \mathrm{T} = \sqrt{\frac{\mathrm{L}}{\mathrm{L/T^2}}} = \sqrt{\mathrm{T}^2} = \mathrm{T}$$

Thus, the $\boxed{\text{dimensions are consistent}}$.

1.2 **(a)** From $x = Bt^2$, we find that $B = x/t^2$. Thus, B has units of $[B] = [x]/[t^2]$ $= \boxed{\mathrm{L/T^2}}$.

(b) If $x = A \sin(2\pi ft)$, then $[A] = [x]/[\sin(2\pi ft)]$

But the sine of an angle is a dimensionless ratio.

Therefore, $[A] = [x] = \boxed{\mathrm{L}}$.

1.3 **(a)** The units of volume, area, and height are: $[V] = \mathrm{L}^3$, $[A] = \mathrm{L}^2$, and $[h] = \mathrm{L}$.

We then observe that $\mathrm{L}^3 = \mathrm{L}^2\mathrm{L}$ or $[V] = [A][h]$. Thus, the equation V

$= Ah$ is $\boxed{\text{dimensionally correct}}$.

(b) $V_{\text{cylinder}} = \pi R^2 h = \left(\pi R^2\right) h = Ah$, where $\boxed{A = \pi R^2}$.

$V_{\text{rectangular box}} = \ell w h = \left(\ell w\right) h = Ah$, where $\boxed{A = \ell w = \text{length} \times \text{width}}$.

1.4 **(a)** In the equation $\frac{1}{2} m v^2 = \frac{1}{2} m v_0^2 + \sqrt{mgh}$,

$[mv^2] = [mv_0^2] = \mathrm{M}\left(\frac{\mathrm{L}}{\mathrm{T}}\right)^2 = \frac{\mathrm{ML}^2}{\mathrm{T}^2}$ while $\left[\sqrt{mgh}\right] = \sqrt{\mathrm{M}\left(\frac{\mathrm{L}}{\mathrm{T}^2}\right)\mathrm{L}} = \frac{\mathrm{M}^{\frac{1}{2}}\mathrm{L}}{\mathrm{T}}$.

Thus, the equation is $\boxed{\text{dimensionally incorrect}}$.

(b) In $v = v_0 + at^2$, $[v] = [v_0] = \frac{\mathrm{L}}{\mathrm{T}}$ but $[at^2] = [a][t^2] = \left(\frac{\mathrm{L}}{\mathrm{T}^2}\right)\left(\mathrm{T}^2\right) = \mathrm{L}$.

Hence, this equation is $\boxed{\text{dimensionally incorrect}}$.

(c) In the equation $ma = v^2$, we see that $[ma] = [m][a] = \mathrm{M}\left(\frac{\mathrm{L}}{\mathrm{T}^2}\right) = \frac{\mathrm{ML}}{\mathrm{T}^2}$,

while $[v^2] = \left(\frac{\mathrm{L}}{\mathrm{T}}\right)^2 = \frac{\mathrm{L}^2}{\mathrm{T}^2}$. Therefore, this equation is also

$\boxed{\text{dimensionally incorrect}}$.

1.5 From the universal gravitation law, the constant G is $G = Fr^2/Mm$. Its units are then

$$[G] = \frac{[F]\left[r^2\right]}{[M][m]} = \frac{\left(\cancel{\mathrm{kg}} \cdot \mathrm{m/s}^2\right)\left(\mathrm{m}^2\right)}{\cancel{\mathrm{kg}} \cdot \mathrm{kg}} = \boxed{\frac{\mathrm{m}^3}{\mathrm{kg} \cdot \mathrm{s}^2}}$$

1.6 **(a)** Solving $KE = p^2/2m$ for the momentum, p, gives $p = \sqrt{2m\left(KE\right)}$

where the numeral 2 is a dimensionless constant. Dimensional analysis gives the units of momentum as:

$$[p] = \sqrt{[m][KE]} = \sqrt{\mathrm{M}\left(\mathrm{M}\cdot\mathrm{L}^2/\mathrm{T}^2\right)} = \sqrt{\mathrm{M}^2\cdot\mathrm{L}^2/\mathrm{T}^2} = \mathrm{M}\left(\mathrm{L}/\mathrm{T}\right)$$

Therefore, in the SI system, the units of momentum are $\boxed{\text{kg}\cdot(\text{m/s})}$.

(b) Note that the units of force are kg · m/s^2 or $[F] = \mathrm{ML/T^2}$. Then, observe that

$$[F][t] = (\mathrm{ML/T^2})\mathrm{T} = \mathrm{M(L/T)} = [p]$$

From this, it follows that force multiplied by time is proportional to momentum: $\boxed{Ft = p}$. (See the impulse-momentum theorem in Topic 6, which says that a constant force F multiplied by a duration of time Δt equals the change in momentum, Δp.)

1.7 The rectangular airstrip's area is its length times its width. The width contains four significant figures while the length contains two. Therefore the answer should be rounded to two significant figures:

$$A = LW = (210\ \text{m})(32.30\ \text{m})$$
$$A = \boxed{6.8\times10^3\ \text{m}^2}\ \text{(rounded to two significant figures)}$$

1.8 The number 15 contains no significant figures after the decimal, so the sum should be rounded to no digits after the decimal:

$$21.4 + 15 + 17.17 + 4.003 = \boxed{58}$$
(rounded to no digits past the decimal)

1.9 The area A of the rectangular room is $A = LW = (9.72 \text{ m})(5.3 \text{ m}) = \boxed{52 \text{ m}^2}$.

1.10 **(a)** Computing $\left(\sqrt{8}\right)^3$ without rounding the intermediate result yields $\left(\sqrt{8}\right)^3 = \boxed{22.6}$ to three significant figures.

(b) Rounding the intermediate result to three significant figures yields $\sqrt{8} = 2.8284 \to 2.83$. Then, we obtain $\left(\sqrt{8}\right)^3 = (2.83)^3 = \boxed{22.7}$ to three significant figures.

(c) $\boxed{\text{The answer 22.6 is more reliable}}$ because rounding in part (b) was carried out too soon.

1.11 **(a)** 78.9 ± 0.2 has $\boxed{\text{3 significant figures}}$ with the uncertainty in the tenths position.

(b) 3.788×10^9 has 4 significant figures $\boxed{\text{4 significant figures}}$

(c) 246×10^{-6} has $\boxed{\text{3 significant figures}}$

(d) $0.003\,2 = 3.2 \times 10^{-3}$ has $\boxed{\text{2 significant figures}}$. The two zeros were originally included only to position the decimal.

1.12 $c = 2.997\,924\,58 \times 10^8$ m/s

(a) Rounded to 3 significant figures: $c = \boxed{3.00 \times 10^8 \text{ m/s}}$

(b) Rounded to 5 significant figures: $c = \boxed{2.997\,9 \times 10^8 \text{ m/s}}$

(c) Rounded to 7 significant figures: $c = \boxed{2.997\,925 \times 10^8 \text{ m/s}}$

1.13 For a length of $L = (2.0 \pm 0.2)$ m and a width of $W = (1.5 \pm 0.1)$ m:

(a) The maximum value for the area, $A = LW$, is found using the maximum values of L and W:

$$A_{\text{max}} = (2.2 \text{ m})(1.6 \text{ m}) = 3.52 \text{ m}^2$$

The minimum area is found using the minimum values of L and W:

$$A_{\text{min}} = (1.8 \text{ m})(1.4 \text{ m}) = 2.52 \text{ m}^2$$

The area using numbers in the middle of the uncertainty range is

$$A_{\text{middle}} = LW = (2.0 \text{ m})(1.5 \text{ m}) = 3.0 \text{ m}^2$$

An uncertainty of about $\pm$ 0.5 m^2 is needed to cover the range from A_{max} to A_{min} so that $A = \boxed{(3.0 \pm 0.5) \text{ m}^2}$.

(b) The perimeter of a rectangle is $P = 2L + 2W = 2(L + W)$. As before, the maximum, minimum, and middle values are:

$$P_{\text{max}} = 2(2.2 \text{ m} + 1.6 \text{ m}) = 7.6 \text{ m}$$

$$P_{\text{min}} = 2(1.8 \text{ m} + 1.4 \text{ m}) = 6.4 \text{ m}$$

$$P_{\text{middle}} = 2(2.0 \text{ m} + 1.5 \text{ m}) = 7.0 \text{ m}$$

An uncertainty of ± 0.6 is needed to cover the range from P_{max} to

P_{min} so that $P = \boxed{(7.0 \pm 0.6)\ \text{m}}$.

1.14 **(a)** $A = \pi r^2 = \pi(10.5\ \text{m} \pm 0.2\ \text{m})^2 = \pi[(10.5\ \text{m})^2 \pm 2(10.5\ \text{m})(0.2\ \text{m}) +$

$(0.2\ \text{m})^2]$. Recognize that the last term in the brackets is insignificant in comparison to the other two. Thus, we have

$$A = \pi[110\ \text{m}^2 \pm 4.2\ \text{m}^2] = \boxed{346\ \text{m}^2 \pm 13\ \text{m}^2}$$

(b) $C = 2\pi r = 2\pi(10.5\ \text{m} \pm 0.2\ \text{m}) = \boxed{66.0\ \text{m} \pm 1.3\ \text{m}}$

1.15 The least accurate dimension of the box has two significant figures. Thus, the volume (product of the three dimensions) will contain only two significant figures.

$$V = \ell \cdot w \cdot h = (29\ \text{cm})(17.8\ \text{cm})(11.4\ \text{cm}) = \boxed{5.9 \times 10^3\ \text{cm}^3}$$

1.16 **(a)** The sum is rounded to $\boxed{797}$ because 756 in the terms to be added has no positions beyond the decimal.

(b) $0.003\,2 \times 356.3 = (3.2 \times 10^{-3}) \times 356.3 = 1.140\,16$ must be rounded to $\boxed{1.1}$ because 3.2×10^{-3} has only two significant figures.

(c) $5.620 \times \pi$ must be rounded to $\boxed{17.66}$ because 5.620 has only four significant figures.

1.17 Use the conversion factor 1 cubitus = 0.445 m to find the basketball player's height:

$$2.00 \text{ m} = 2.00 \text{ m}\left(\frac{1 \text{ cubitus}}{0.445 \text{ m}}\right) = \boxed{4.49 \text{ cubiti}}$$

1.18 Use the conversion factor 1 m = 3.281 ft to find

$$1\ 420 \text{ ft}^2 = 1\ 420 \text{ ft}^2\left(\frac{1 \text{ m}}{3.281 \text{ ft}}\right)^2 = \boxed{132 \text{ m}^2}$$

1.19 $$d = (250\,000 \text{ mi})\left(\frac{5\ 280 \text{ ft}}{1.000 \text{ mi}}\right)\left(\frac{1 \text{ fathom}}{6 \text{ ft}}\right) = \boxed{2\times 10^8 \text{ fathoms}}$$

The answer is limited to one significant figure because of the accuracy to which the conversion from fathoms to feet is given.

1.20

$$v = \frac{\ell}{t} = \frac{186 \cancel{\text{ furlongs}}}{1 \cancel{\text{ fortnight}}}\left(\frac{1 \cancel{\text{ fortnight}}}{14 \cancel{\text{ days}}}\right)\left(\frac{1 \cancel{\text{ days}}}{8.64\times 10^4 \text{ s}}\right)\left(\frac{220 \cancel{\text{ yds}}}{1 \cancel{\text{ furlongs}}}\right)\left(\frac{3 \cancel{\text{ ft}}}{1 \cancel{\text{ yd}}}\right)\left(\frac{100 \text{ cm}}{3.281 \cancel{\text{ ft}}}\right)$$

giving $v = \boxed{3.09 \text{ cm/s}}$

1.21 $$6.00 \text{ firkins} = 6.00 \cancel{\text{ firkins}}\left(\frac{9 \cancel{\text{ gal}}}{1 \cancel{\text{ firkin}}}\right)\left(\frac{3.786 \cancel{\text{ L}}}{1 \cancel{\text{ gal}}}\right)\left(\frac{10^3 \cancel{\text{ cm}^3}}{1 \cancel{\text{ L}}}\right)\left(\frac{1 \text{ m}^3}{10^6 \cancel{\text{ cm}^3}}\right)$$

$$= \boxed{0.204 \text{ m}^3}$$

1.22 **(a)** $$\ell = (348 \text{ mi})\left(\frac{1.609 \text{ km}}{1.000 \text{ mi}}\right) = \boxed{5.60\times 10^2 \text{ km}} = \boxed{5.60\times 10^5 \text{ m}}$$

$$= \boxed{5.60\times 10^7 \text{ cm}}$$

(b) $$h = (1\,612 \text{ ft})\left(\frac{1.609 \text{ km}}{5\ 280 \text{ ft}}\right) = \boxed{0.491\ 2 \text{ km}} = \boxed{491.2 \text{ m}}$$

$$= \boxed{4.912\times 10^4 \text{ cm}}$$

(c) $h = (20\ 320\ \text{ft})\left(\dfrac{1.609\ \text{km}}{5\ 280\ \text{ft}}\right) = \boxed{6.192\ \text{km}} = \boxed{6.192\times 10^{3}\ \text{m}}$

$= \boxed{6.192\times 10^{5}\ \text{cm}}$

(d) $d = (8\ 200\ \text{ft})\left(\dfrac{1.609\ \text{km}}{5\ 280\ \text{ft}}\right) = \boxed{2.499\ \text{km}} = \boxed{2.499\times 10^{3}\ \text{m}}$

$= \boxed{2.499\times 10^{5}\ \text{cm}}$

In (a), the answer is limited to three significant figures because of the accuracy of the original data value, 348 miles. In (b), (c), and (d), the answers are limited to four significant figures because of the accuracy to which the kilometers-to-feet conversion factor is given.

1.23 $v = 38.0\dfrac{\cancel{\text{m}}}{\cancel{\text{s}}}\left(\dfrac{1\ \cancel{\text{km}}}{10^{3}\ \cancel{\text{m}}}\right)\left(\dfrac{1\ \text{mi}}{1.609\ \cancel{\text{km}}}\right)\left(\dfrac{3\ 600\ \cancel{\text{s}}}{1\ \text{h}}\right) = 85.0\ \text{mi/h}$

$\boxed{\text{Yes, the driver is exceeding the speed limit by 10.0 mi/h}}$.

1.24 $efficiency = 25.0\dfrac{\cancel{\text{mi}}}{\cancel{\text{gal}}}\left(\dfrac{1\ \text{km}}{0.621\ \cancel{\text{mi}}}\right)\left(\dfrac{1\ \cancel{\text{gal}}}{3.786\ \text{L}}\right) = \boxed{10.6\ \text{km/L}}$

1.25 **(a)** $r = \dfrac{diameter}{2} = \dfrac{5.36\ \text{in}}{2}\left(\dfrac{2.54\ \text{cm}}{1\ \text{in}}\right) = \boxed{6.81\ \text{cm}}$

(b) $A = 4\pi r^{2} = 4\pi\,(6.81\ \text{cm})^{2} = \boxed{5.83\times 10^{2}\ \text{cm}^{2}}$

(c) $V = \dfrac{4}{3}\pi r^{3} = \dfrac{4}{3}\pi\left(6.81\ \text{cm}\right)^{3} = \boxed{1.32\times 10^{3}\ \text{cm}^{3}}$

1.26 $rate = \left(\dfrac{1}{32}\ \dfrac{\cancel{\text{in}}}{\cancel{\text{day}}}\right)\left(\dfrac{1\ \cancel{\text{day}}}{24\ \cancel{\text{h}}}\right)\left(\dfrac{1\ \cancel{\text{h}}}{3\ 600\ \text{s}}\right)\left(\dfrac{2.54\ \cancel{\text{cm}}}{1.00\ \cancel{\text{in}}}\right)\left(\dfrac{10^{9}\ \text{nm}}{10^{2}\ \cancel{\text{cm}}}\right) = \boxed{9.2\ \text{nm/s}}$

This means that the proteins are assembled at a rate of many layers of atoms each second!

1.27 $$c=\left(3.00\times10^{8}\ \frac{\cancel{\text{m}}}{\cancel{\text{s}}}\right)\left(\frac{3\,600\ \cancel{\text{s}}}{1\ \text{h}}\right)\left(\frac{1\ \cancel{\text{km}}}{10^{3}\ \cancel{\text{m}}}\right)\left(\frac{1\ \text{mi}}{1.609\ \cancel{\text{km}}}\right)=\boxed{6.71\times10^{8}\ \text{mi/h}}$$

1.28
$$\begin{aligned}\text{Volume of house}&=(50.0\ \text{ft})(26\ \text{ft})(8.0\ \text{ft})\left(\frac{2.832\times10^{-2}\ \text{m}^3}{1\ \text{ft}^3}\right)\\&=\boxed{2.9\times10^{2}\ \text{m}^3}=(2.9\times10^{2}\ \text{m}^3)\left(\frac{100\ \text{cm}}{1\ \text{cm}}\right)^3\\&=\boxed{2.9\times10^{8}\ \text{cm}^3}\end{aligned}$$

1.29
$$\text{Volume}=\left(25.0\ \cancel{\text{acre}}\ \cancel{\text{ft}}\right)\left(\frac{1\ \text{m}}{3.281\ \cancel{\text{ft}}}\right)\left[\left(\frac{43\,560\ \cancel{\text{ft}}^2}{1\ \cancel{\text{acre}}}\right)\left(\frac{1\ \text{m}}{3.281\ \cancel{\text{ft}}}\right)^2\right]$$
$$=\boxed{3.08\times10^{4}\ \text{m}^3}$$

1.30

$$\begin{aligned}\text{Volume of pyramid}&=\frac{1}{3}(\text{area of base})(\text{height})\\&=\frac{1}{3}[(13.0\ \text{acres})(43560\ \text{ft}^2/\text{acre})](481\ \text{ft})=9.08\times10^{7}\ \text{ft}^3\\&=(9.08\times10^{3}\ \text{ft}^3)\left(\frac{2.832\times10^{-2}\ \text{m}^3}{1\ \text{ft}^3}\right)=\boxed{2.57\times10^{6}\ \text{m}^3}\end{aligned}$$

1.31 Volume of cube $=L^3=1$ quart (Where L = length of one side of the cube.)

Thus, $$L^3=\left(1\ \cancel{\text{quart}}\right)\left(\frac{1\ \cancel{\text{gallon}}}{4\ \cancel{\text{quarts}}}\right)\left(\frac{3.786\ \cancel{\text{liter}}}{1\ \cancel{\text{gallon}}}\right)\left(\frac{1000\ \text{cm}^3}{1\ \cancel{\text{liter}}}\right)=947\ \text{cm}^3$$

and $L = \sqrt[3]{947 \text{ cm}^3} = \boxed{9.82 \text{ cm}}$

1.32 We estimate that the length of a step for an average person is about 18 inches, or roughly 0.5 m.

Then, an estimate for the number of steps required to travel a distance equal to the circumference of the Earth would be

$$N = \frac{\text{Circumference}}{\text{Step Length}} = \frac{2\pi R_E}{\text{Step Length}} = \frac{2\pi(6.38 \times 10^6 \text{ m})}{0.5 \text{ m/step}} \approx 8 \times 10^7 \text{ steps}$$

or $\boxed{N \sim 10^3 \text{ steps}}$

1.33 We assume an average respiration rate of about 10 breaths/minute and a typical life span of 70 years. Then, an estimate of the number of breaths an average person would take in a lifetime is

$$n = \left(10 \frac{\text{breaths}}{\cancel{\text{min}}}\right)(70 \cancel{\text{ yr}})\left(\frac{3.156 \times 10^7 \cancel{\text{ s}}}{1 \cancel{\text{ yr}}}\right)\left(\frac{1 \cancel{\text{ min}}}{60 \cancel{\text{ s}}}\right) = 4 \times 10^8 \text{ breaths}$$

or $\boxed{n \sim 10^8 \text{ breaths}}$

1.34 We assume that the average person catches a cold twice a year and is sick an average of 7 days (or 1 week) each time. Thus, on average, each person is sick for 2 weeks out of each year (52 weeks). The probability that a particular person will be sick at any given time equals the percentage of time that person is sick, or

$$probability\ of\ sickness = \frac{2\text{ weeks}}{52\text{ weeks}} = \frac{1}{26}$$

The population of the Earth is approximately 7 billion. The number of people expected to have a cold on any given day is then

$$Number\ sick = (population)(probability\ of\ sickness) = (7\times10^{9})\left(\frac{1}{26}\right) = 3\times10^{8}\text{ or }\boxed{10^{8}}$$

1.35 Earth's current population is about 7.3 billion people. If six people can stand within a 1-m^2 area, the total area required to fit all 7.3 billion people is:

$$A_{\text{people}} = 7.3\times10^{9}\text{ people}\left(\frac{1\text{ m}^2}{6\text{ people}}\right) = 1.2\times10^{9}\text{ m}^2$$

The required percentage of the habitable part of Earth's surface is:

$$\text{Occupied percent} = \left(\frac{1.2\times10^{9}\text{ m}^2}{60\times10^{12}\text{ m}^2}\right)100\% = \boxed{2\times10^{-3}\%}$$

1.36 **(a)** $V_{\text{cell}} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(1.0\times10^{-6}\text{ m})^3 = \boxed{4.2\times10^{-18}\text{ m}^3}$

(b) Consider your body to be a cylinder having a radius of about 6 inches (or 0.15 m) and a height of about 1.5 meters. Then, its volume is

$$V_{\text{body}} = Ah = (\pi r^2)h = \pi(0.15\text{ m})^2(1.5\text{ m}) = 0.11\text{ m}^3\text{ or }\boxed{\sim10^{-1}\text{ m}^3}$$

(c) The estimate of the number of cells in the body is then

$$n = \frac{V_{\text{body}}}{V_{\text{cell}}} = \frac{0.11 \text{ m}^3}{4.2 \times 10^{-18} \text{ m}^3} = 2.6 \times 10^{16} \text{ or } \boxed{\sim 10^{16}}$$

1.37 A reasonable guess for the diameter of a tire might be 3 ft, with a circumference ($C = 2\pi r = \pi D$ = distance traveled per revolution) of about 9 ft. Thus, the total number of revolutions the tire might make is

$$n = \frac{\text{total distance traveled}}{\text{distance per revolution}} = \frac{(50\,000 \text{ mi})(5\,280 \text{ ft/mi})}{9 \text{ ft/rev}} = 3 \times 10^7 \text{ rev, or} \boxed{\sim 10^7 \text{ rev}}$$

1.38 An average body mass is about 70 kg and, taking the center of the given range, approximately 2% or 1.4 kg of this mass is made of microorganisms. If there are 100 trillion microorganisms in a human body, the average mass of a microorganism is about

$$m_{\text{microorganism}} \approx \frac{1.4 \text{ kg}}{100 \times 10^{12} \text{ microorganisms}} \approx \boxed{1 \times 10^{-14} \text{ kg}}$$

1.39 The x coordinate is found as $x = r\cos\theta = (2.5 \text{ m})\cos 35° = \boxed{2.0 \text{ m}}$

and the y coordinate $y = r\sin\theta = (2.5 \text{ m})\sin 35° = \boxed{1.4 \text{ m}}$

1.40 The x distance out to the fly is 2.0 m and the y distance up to the fly is 1.0 m. Thus, we can use the Pythagorean theorem to find the distance from the origin to the fly as

$$d = \sqrt{x^2 + y^2} = \sqrt{(2.0 \text{ m})^2 + (1.0 \text{ m})^2} = \boxed{2.2 \text{ m}}$$

1.41 The distance from the origin to the fly is r in polar coordinates, and this was found to be 2.2 m in Problem 40. The angle θ is the angle between r and the horizontal reference line (the x axis in this case). Thus, the angle can be found as

$$\tan\theta = \frac{y}{x} = \frac{1.0 \text{ m}}{2.0 \text{ m}} = 0.50 \quad \text{and} \quad \theta = \tan^{-1}(0.50) = 27°$$

The polar coordinates are $\boxed{r = 2.2 \text{ m and } \theta = 27°}$

1.42 The x distance between the two points is $|\Delta x| = |x_2 - x_1| = |-3.0 \text{ cm} - 5.0 \text{ cm}| = 8.0$ cm and the y distance between them is $|\Delta y| = |y_2 - y_1| = |-3.0 \text{ cm} - 4.0 \text{ cm}| = 1.0$ cm. The distance between them is found from the Pythagorean theorem:

$$d = \sqrt{|\Delta x|^2 + |\Delta y|^2} = \sqrt{(8.0 \text{ cm})^2 + (1.0 \text{ cm})^2} = \sqrt{65 \text{ cm}^2} = \boxed{8.1 \text{ cm}}$$

1.43 Refer to the Figure given in the solution to Problem 1.44 below. The Cartesian coordinates for the two given points are:

$x_1 = r_1 \cos\theta_1 = (2.00 \text{ m}) \cos 50.0° = 1.29 \text{ m}$

$y_1 = r_1 \sin\theta_1 = (2.00 \text{ m}) \sin 50.0° = 1.53 \text{ m}$

$x_2 = r_2 \cos\theta_2 = (5.00 \text{ m}) \cos(-50.0°) = 3.21 \text{ m}$

$y_2 = r_2 \sin\theta_2 = (5.00 \text{ m}) \sin(-50.0°) = 3.83 \text{ m}$

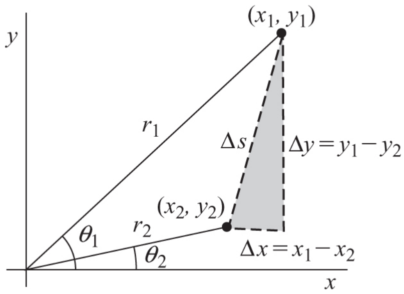
The distance between the two points is then:

$$\Delta s = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(1.29 \text{ m} - 3.21 \text{ m})^2 + (1.53 \text{ m} + 3.83 \text{ m})^2} = \boxed{5.69 \text{ m}}$$

1.44 Consider the Figure shown below. The Cartesian coordinates for the two points are:

$x_1 = r_1 \cos\theta_1$ $\qquad x_2 = r_2 \cos\theta_2$

$y_1 = r_1 \sin\theta_1$ $\qquad y_2 = r_2 \sin\theta_2$



The distance between the two points is the length of the hypotenuse of the shaded triangle and is given by

$$\Delta s = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

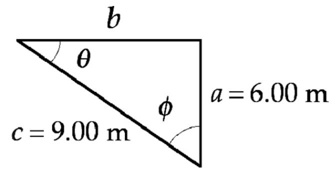
or

$$\begin{aligned}\Delta s &= \sqrt{(r_1^2\cos^2\theta_1 + r_2^2\cos^2\theta_2 - 2r_1r_2\cos\theta_1\cos\theta_2) + (r_1^2\sin^2\theta_1 + r_2^2\sin^2\theta_2 - 2r_1r_2\sin\theta_1\sin\theta_2)} \\ &= \sqrt{r_1^2(\cos^2\theta_1 + \sin^2\theta_1) + r_2^2(\cos^2\theta_2 + \sin^2\theta_2) - 2r_1r_2(\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2)}\end{aligned}$$

Applying the identities $\cos^2\theta + \sin^2\theta = 1$ and $\cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2 = \cos(\theta_1 - \theta_2)$, this reduces to

$$\boxed{\Delta s = \sqrt{r_1^2 + r_2^2 - 2r_1r_2(\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2)}} = \boxed{\sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_1 - \theta_2)}}$$

1.45 **(a)** With $a = 6.00$ m and b being two sides of this right triangle having hypotenuse $c = 9.00$ m, the Pythagorean theorem gives the unknown side as $b=\sqrt{c^2-a^2}=\sqrt{(9.00\text{ m})^2-(6.00\text{ m})^2}=\boxed{6.71\text{ m}}$.

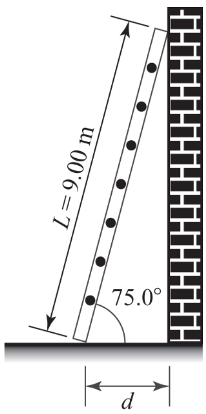


(b) $\tan\theta=\dfrac{a}{b}=\dfrac{6.00\text{ m}}{6.71\text{ m}}=\boxed{0.894}$

(c) $\sin\phi=\dfrac{b}{c}=\dfrac{6.71\text{ m}}{9.00\text{ m}}=\boxed{0.746}$

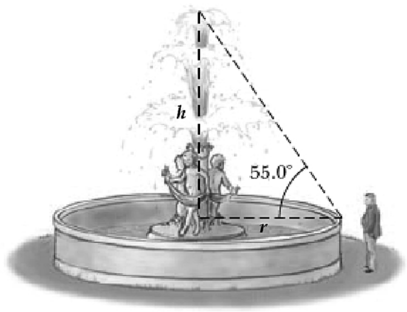
1.46 From the diagram, $\cos(75.0°)=d/L$

Thus, $d=L\cos(75.0°)=(9.00\text{ m})\cos(75.0°)=\boxed{2.33\text{ m}}$



1.47 The circumference of the fountain is $C=2\pi r$, so the radius is

$$r=\frac{C}{2\pi}=\frac{15.0\text{ m}}{2\pi}=2.39\text{ m}$$



Thus, $\tan(55.0°) = \frac{h}{r} = \frac{h}{2.39\text{ m}}$ which gives

$h = (2.39\text{ m})\tan(55.0°) = \boxed{3.41\text{ m}}$.

1.48 **(a)** $\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$ so, opposite side = (3.00 m) sin (30.0°) = $\boxed{1.50\text{ m}}$

(b) $\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$ so, adjacent side = (3.00 m) cos (30.0°) = $\boxed{2.60\text{ m}}$

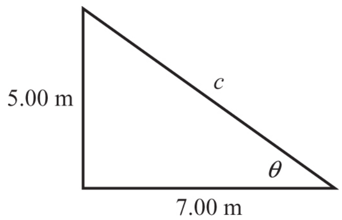
1.49 **(a)** The side opposite $\theta = \boxed{3.00}$ **(b)** The side adjacent to $\phi = \boxed{3.00}$

(c) $\cos\theta = \frac{4.00}{5.00} = \boxed{0.800}$ **(d)** $\sin\phi = \frac{4.00}{5.00} = \boxed{0.800}$

(e) $\tan\phi = \frac{4.00}{3.00} = \boxed{1.33}$

1.50 Using the diagram below, the Pythagorean theorem yields

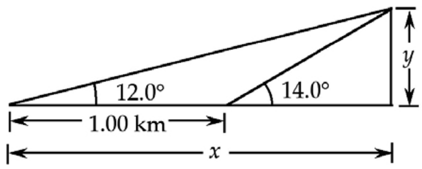
$$c = \sqrt{(5.00\text{ m})^2 + (7.00\text{ m})^2} = \boxed{8.60\text{ m}}$$



1.51 From the diagram given in Problem 1.50 above, it is seen that

$$\tan\theta = \boxed{\frac{5.00}{7.00}} = 0.714 \quad \text{and} \quad \theta = \tan^{-1}(0.714) = \boxed{35.5°}$$

1.52 Use the diagram below to equate two expressions for the mountain's height (units and significant digits have been suppressed for clarity):



$$y = (x - 1)\tan(14.0) = (x)\tan(12.0°)$$

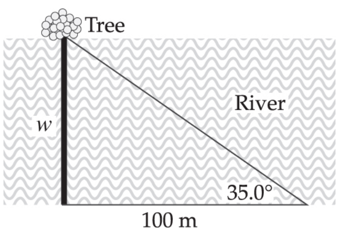
Solve for the unknown distance x to find

$$\begin{aligned}(x-1)\tan(14.0°) &= x\tan(12.0°)\\ x(\tan 14° - \tan 12°) &= \tan 14°\\ x &= \frac{\tan 14°}{\tan 14° - \tan 12°} = 6.78 \text{ km}\end{aligned}$$

Substitute the known value of x into $y = (x)\tan(12.0°)$ to find the mountain's height:

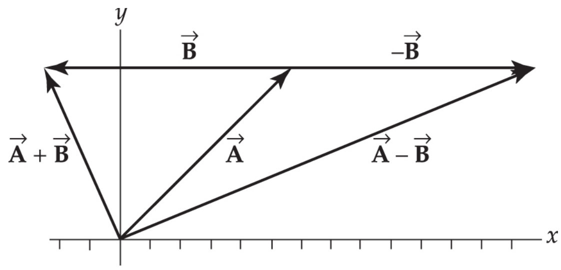
$$\begin{aligned}y &= x\tan(12°) = (6.78 \text{ km})\tan(12°)\\ &= 1.44 \text{ km} = \boxed{1.44 \times 10^{3} \text{ m}}\end{aligned}$$

1.53 Using the sketch below:



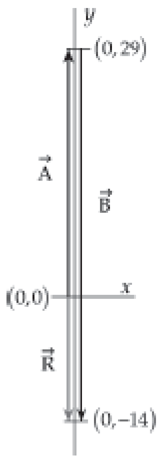
$$\frac{w}{100\text{ m}} = \tan 35.0°\text{, or } w = (100\text{ m})\tan 35.0° = \boxed{70.0\text{ m}}$$

1.54 **(a)** Using graphical methods, place the tail of vector $\vec{\mathbf{B}}$ at the head of vector $\vec{\mathbf{A}}$. The new vector $\vec{\mathbf{A}}+\vec{\mathbf{B}}$ has a magnitude of $\boxed{6.1\text{ units at } 113°}$ from the positive x-axis.



(b) The vector difference $\vec{\mathbf{A}}-\vec{\mathbf{B}}=\vec{\mathbf{A}}+(-\vec{\mathbf{B}})$ is found by placing the negative of vector $\vec{\mathbf{B}}$ (a vector of the same magnitude as $\vec{\mathbf{B}}$, but opposite direction) at the head of vector $\vec{\mathbf{A}}$. The resultant vector $\vec{\mathbf{A}}-\vec{\mathbf{B}}$ has magnitude $\boxed{15\text{ units at } 23°}$ from the positive x-axis.

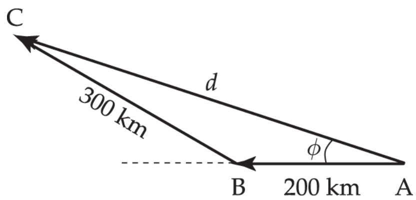
1.55 We are given that $\vec{\mathbf{R}}=\vec{\mathbf{A}}+\vec{\mathbf{B}}$. When two vectors are added graphically, the second vector is positioned with its tail at the tip of the first vector. The resultant then runs from the tail of the first vector to the tip of the second vector. In this case, vector $\vec{\mathbf{A}}$ will be positioned with its tail at the origin and its tip at the point (0, 29). The resultant is then drawn, starting at the origin (tail of first vector) and going 14 units in the negative y-direction to the point (0, –14). The second vector, $\vec{\mathbf{B}}$, must then start from the tip of $\vec{\mathbf{A}}$ at point (0, 29) and end on the tip of $\vec{\mathbf{R}}$ at point (0, −14) as shown in the sketch below. From this, it is seen that



$\boxed{\vec{\mathbf{B}} \text{ is 43 units in the negative } y\text{-direction}}$

1.56 **(a)** The distance d from A to C is $d = \sqrt{x^2 + y^2}$ where $x = 200$ km + (300 km)cos 30.0° = 460 km and $y = 0$ + (300 km)sin 30.0° = 150 km.

$$\therefore \; d = \sqrt{(460 \text{ km})^2 + (150 \text{ km})^2} = \boxed{484 \text{ km}}$$



(b) $$\phi = \tan^{-1}\frac{y}{x} = \tan^{-1}\left(\frac{150 \text{ km}}{460 \text{ km}}\right) = \boxed{18.1° \text{ N of W}}$$

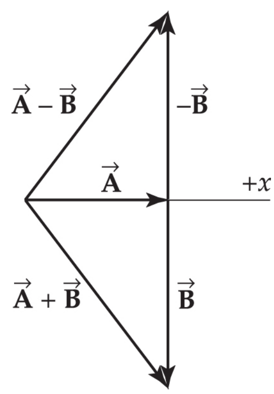
(c) Because of the curvature of the Earth, the plane doesn't travel along straight lines. Thus, the answer computed above is only approximately correct.

1.57 **(a)** In your vector diagram, place the tail of vector $\vec{\mathbf{B}}$ at the tip of vector $\vec{\mathbf{A}}$. The vector sum, $\vec{\mathbf{A}} + \vec{\mathbf{B}}$, is then found as shown in the vector diagram and should be

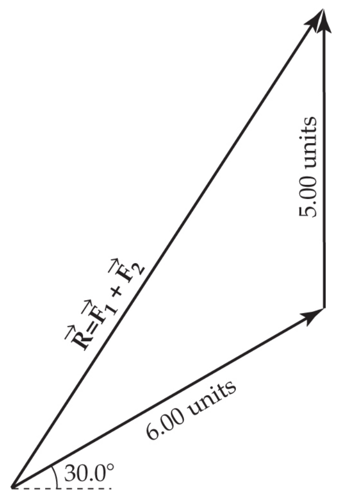
$$\boxed{\vec{\mathbf{A}}+\vec{\mathbf{B}}=5.0 \text{ units at } -53^\circ}$$

(b) To find the vector difference $\vec{\mathbf{A}}-\vec{\mathbf{B}}=\vec{\mathbf{A}}+(-\vec{\mathbf{B}})$, form the vector $-\vec{\mathbf{B}}$ (same magnitude as $\vec{\mathbf{B}}$, opposite direction) and add it to vector $\vec{\mathbf{A}}$ as shown in the diagram. You should find that

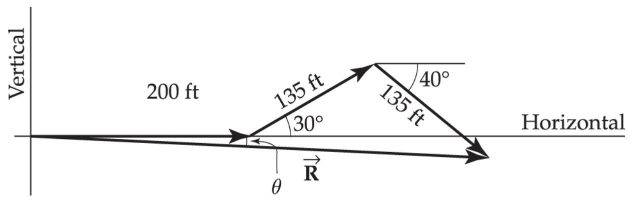
$$\boxed{\vec{\mathbf{A}}-\vec{\mathbf{B}}=5.0 \text{ units at } +53^\circ}$$



1.58 Find the resultant $\vec{\mathbf{R}}=\vec{\mathbf{F}}_1+\vec{\mathbf{F}}_2$ graphically by placing the tail of $\vec{\mathbf{F}}_2$ at the head of $\vec{\mathbf{F}}_1$ in a scale drawing as shown below. The resultant is the vector drawn from the tail of $\vec{\mathbf{F}}_1$ to the head of $\vec{\mathbf{F}}_2$ to close up the triangle. Measuring the length and orientation of this vector shows that $\vec{\mathbf{R}}=\boxed{9.5 \text{ units at } 57^\circ \text{ above the } +x\text{-axis}}$.



1.59 Your sketch should be drawn to scale, and be similar to that pictured below. The length of $\vec{\mathbf{R}}$ and the angle θ can be measured to find, with use of your scale factor, the magnitude and direction of the resultant displacement. The result should be $\boxed{\text{approximately 421 ft 3° below the horizontal}}$.



1.60 **(a)** The x-component is $x = (24.0\text{ m})\cos(56.0°) = \boxed{13.4\text{ m}}$.

(b) The y-component is $y = (24.0\text{ m})\sin(56.0°) = \boxed{19.9\text{ m}}$.

1.61 **(a)** The vector's magnitude is

$$\begin{aligned} A = \left|\vec{\mathbf{A}}\right| &= \sqrt{A_x^2 + A_y^2} \\ &= \sqrt{(-5.00\text{ m})^2 + (9.00\text{ m})^2} \\ &= \boxed{10.3\text{ m}} \end{aligned}$$

(b) The vector is in the second quadrant so its direction is

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) + 180° = \tan^{-1}\left(\frac{9.00\text{ m}}{-5.00\text{ m}}\right) + 180°$$
$$= \boxed{119°}$$

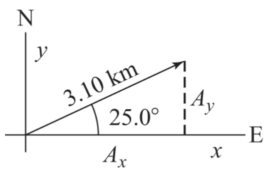
1.62 The person undergoes a displacement

$\vec{\mathbf{A}} = 3.10\text{ km}$ at $25.0°$ north of east. Choose a coordinate system with origin at the starting point, positive x-direction oriented eastward, and positive y-direction oriented northward.

Then the components of her displacement are

$$A_x = A\cos\theta = (3.10\text{ km})\cos 25.0° = \boxed{2.81\text{ km eastward}}$$

and $A_y = A\sin\theta = (3.10\text{ km})\sin 25.0° = \boxed{1.31\text{ km northward}}$

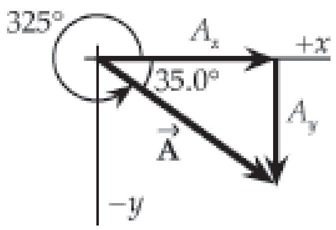


1.63 The x- and y-components of vector $\vec{\mathbf{A}}$ are its projections on lines parallel to the x- and y-axes, respectively, as shown in the sketch. The magnitude of these components can be computed using the sine and cosine functions as shown below:

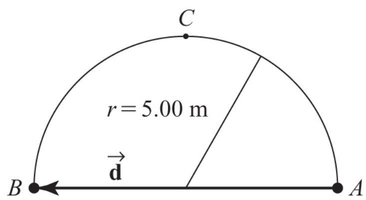
$$A_x = \left|\vec{\mathbf{A}}\right|\cos 325° = +\left|\vec{\mathbf{A}}\right|\cos 35° = (35.0)\cos 35° = \boxed{28.7\text{ units}}$$

and

$$A_y = \left|\vec{\mathbf{A}}\right|\sin 325° = -\left|\vec{\mathbf{A}}\right|\sin 35° = -(35.0)\sin 35° = \boxed{-20.1\text{ units}}$$



1.64 **(a)** The skater's displacement vector, $\vec{\mathbf{d}}$, extends in a straight line from her starting point A to the end point B. When she has coasted half way around a circular path as shown in the sketch below, the displacement vector coincides with the diameter of the circle and has magnitude $|\vec{\mathbf{d}}| = 2r = 2(5.00 \text{ m}) = \boxed{10.0 \text{ m}}$.



(b) The actual distance skated, s, is one half the circumference of the circular path of radius r. Thus $s = \frac{1}{2}(2\pi r) = \pi(5.00 \text{ m}) = \boxed{15.7 \text{ m}}$.

(c) When the skater skates all the way around the circular path, her end point, B, coincides with the start point, A. Thus, the displacement vector has zero length, or $|\vec{\mathbf{d}}| = \boxed{0}$.

1.65 **(a)** Her net x (east-west) displacement is –3.00 + 0 + 6.00 = +3.00 blocks, while her net y (north-south) displacement is 0 + 4.00 + 0 = +4.00 blocks. The magnitude of the resultant displacement is

$$R = \sqrt{(\Sigma x)^2 + (\Sigma y)^2} = \sqrt{(3.00)^2 + (4.00)^2} = 5.00 \text{ blocks}$$

and the angle the resultant makes with the x-axis (eastward direction) is

$$\theta = \tan^{-1}\left(\frac{\Sigma y}{\Sigma x}\right) = \tan^{-1}\left(\frac{4.00}{3.00}\right) = \tan^{-1}(1.33) = 53.1°$$

The resultant displacement is then $\boxed{5.00 \text{ blocks at } 53.1° \text{ N of E}}$.

(b) The total distance traveled is $3.00 + 4.00 + 6.00 = \boxed{13.0 \text{ blocks}}$.

1.66 Let $\vec{\mathbf{A}}$ be the vector corresponding to the 10.0 yd run, $\vec{\mathbf{B}}$ to the 15.0 yd run, and $\vec{\mathbf{C}}$ to the 50.0 yd pass. Also, we choose a coordinate system with the $+y$-direction downfield, and the $+x$-direction toward the sideline to which the player runs.

The components of the vectors are then

$A_x = 0$	$A_y = -10.0$ yds
$B_x = 15.0$ yds	$B_y = 0$
$C_x = 0$	$C_y = +50.0$ yds

From these, $R_x = \Sigma x = 15.0$ yds, and $R_y = \Sigma y = 40.0$ yds,

and $$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(15.0 \text{ yds})^2 + (40.0 \text{ yds})^2} = \boxed{42.7 \text{ yards}}$$

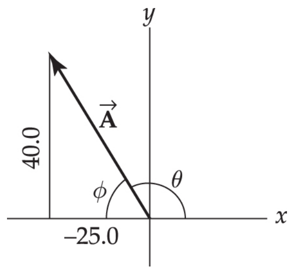
1.67 $A_x = -25.0$ $A_y = 40.0$

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(-25.0)^2 + (40.0)^2} = 47.2 \text{ units}$$

From the triangle, we find that

$$\phi = \tan^{-1}\left(\frac{A_y}{|A_x|}\right) = \tan^{-1}\left(\frac{|40.0|}{25.0}\right) = 58.0°\text{, so } \theta = 180° - \phi = 122°$$

Thus, $\boxed{\vec{\mathbf{A}} = 47.2 \text{ units at } 122° \text{ counterclockwise from the } +x\text{-axis}}$



1.68 Choose the positive x-direction to be eastward and positive y as northward. Then, the components of the resultant displacement from Dallas to Chicago are

$$R_x = \Sigma x = (730 \text{ mi})\cos 5.00° - (560 \text{ mi})\sin 21.0° = 527 \text{ mi}$$

and $R_y = \Sigma y = (730 \text{ mi})\sin 5.00° + (560 \text{ mi})\cos 21.0°$

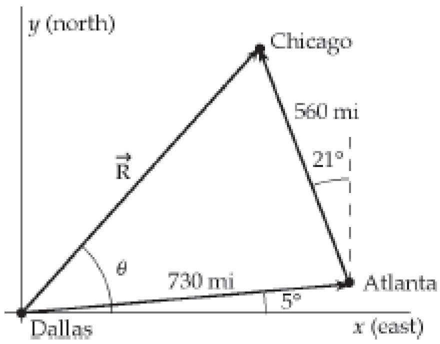
$$= 586 \text{ mi}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(527 \text{ mi})^2 + (586 \text{ mi})^2} = 788 \text{ mi}$$

$$\theta = \tan^{-1}\left(\frac{\Sigma y}{\Sigma x}\right) = \tan^{-1}(1.11) = 48.0°$$

Thus, the displacement from Dallas to Chicago is

$\vec{\mathbf{R}} = \boxed{788 \text{ mi at } 48.0° \text{ N of E}}$



1.69 After 3.00 h moving at 41.0 km/h, the hurricane is 123 km at 60.0° N of W from the island. In the next 1.50 h, it travels 37.5 km due north. The components of these two displacements are:

Displacement	x-component (eastward)	y-component (northward)
123 km	–61.5 km	+107 km
37.5 km	0	+37.5 km
Resultant	**–61.5 km**	**144 km**

Therefore, the eye of the hurricane is now

$$R = \sqrt{(-61.5\text{ km})^2 + (144\text{ km})^2} = \boxed{157\text{ km from the island}}$$

1.70 **(a)** $F_1 = 120\text{ N}$

$F_{1x} = (120\text{ N})\cos 60.0° = 60.0\text{ N}$

$F_{1y} = (120\text{ N})\sin 60.0° = 104\text{ N}$

$F_2 = 80.0\text{ N}$

$F_{2x} = -(80\text{ N})\cos 75.0° = -20.7\text{ N}$

$F_{2y} = (80.0\text{ N})\sin 75.0° = 77.3\text{ N}$

$$F_R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = \sqrt{(39.3\text{ N})^2 + (181\text{ N})^2} = 185\text{ N}$$

and $\theta = \tan^{-1}\left(\dfrac{181\text{ N}}{39.3\text{ N}}\right) = \tan^{-1}(4.61) = 77.8°$

The resultant force is $\vec{\mathbf{F}}_R = \boxed{185\text{ N at }77.8°\text{ from the }x\text{-axis}}$

(b) To have zero net force on the mule, the resultant above must be cancelled by a force equal in magnitude and oppositely directed. Thus, the required force is $\boxed{185\text{ N at }258°\text{ from the }x\text{-axis}}$

1.71 The components of the displacements $\vec{\mathbf{a}}$, $\vec{\mathbf{b}}$, and $\vec{\mathbf{c}}$ are

$a_x = a\cdot\cos 30.0° = +152\text{ km}$

$b_x = b\cdot\cos 110° = -51.3\text{ km}$

$c_x = c\cdot\cos 180° = -190\text{ km}$

and $a_y = a\cdot\sin 30.0° = +87.5\text{ km}$

$b_y = b\cdot\sin 110° = +141\text{ km}$

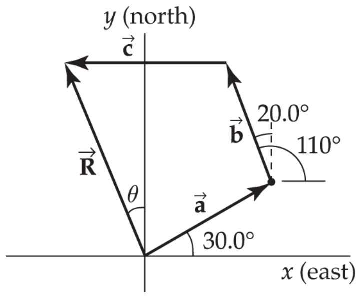
$c_y = c\cdot\sin 180° = 0$

Thus, $R_x = a_x + b_x + c_x = -89.3\text{ km}$, and $R_y = a_y + b_y + c_y = 229\text{ km}$

so $R = \sqrt{R_x^2 + R_y^2} = 246 \text{ km}$, and

$\theta = \tan^{-1}\left(|R_x|/R_y\right) = \tan^{-1}(0.390) = 21.3°$

City C is $\boxed{246 \text{ km at } 21.3° \text{ W of N}}$ from the starting point.



1.72 **(a)** $1\,\dfrac{\text{mi}}{\text{h}} = \left(1\,\dfrac{\text{mi}}{\text{h}}\right)\left(\dfrac{1.609 \text{ km}}{1 \text{ mi}}\right) = \boxed{1.609\,\dfrac{\text{km}}{\text{h}}}$

(b) $v_{\text{max}} = 55\,\dfrac{\text{mi}}{\text{h}} = \left(55\,\dfrac{\text{mi}}{\text{h}}\right)\left(\dfrac{1.609 \text{ km/h}}{1 \text{ mi/h}}\right) = \boxed{88\,\dfrac{\text{km}}{\text{h}}}$

(c) $\Delta v_{\text{max}} = 65\,\dfrac{\text{mi}}{\text{h}} - 55\,\dfrac{\text{mi}}{\text{h}} = \left(10\,\dfrac{\text{mi}}{\text{h}}\right)\left(\dfrac{1.609 \text{ km/h}}{1 \text{ mi/h}}\right) = \boxed{16\,\dfrac{\text{km}}{\text{h}}}$

1.73 The term s has dimensions of L, a has dimensions of LT^{-2}, and t has dimensions of T. Therefore, the equation, $s = ka^m\, t^n$ with k being dimensionless, has dimensions of

$$L = (LT^{-2})^m\, (T)^n \qquad \text{or} \qquad L^1T^0 = L^m\, T^{n-2m}$$

The powers of L and T must be the same on each side of the equation.

Therefore, $L^1 = L^m$ and $\boxed{m = 1}$

Likewise, equating powers of T, we see that $n - 2m = 0$, or $\boxed{n = 2m = 2}$

$\boxed{\text{Dimensional analysis cannot determine the value of } k}$, a dimensionless constant.

1.74 **(a)** The rate of filling in gallons per second is

$$rate = \frac{30.0\text{ gal}}{7.00\text{ min}}\left(\frac{1\text{ min}}{60\text{ s}}\right) = \boxed{7.14\times10^{-2}\text{ gal/s}}$$

(b) Note that $1\text{ m}^3 = (10^2\text{ cm})^3 = (10^6\text{ cm}^3)\left(\frac{1\text{ L}}{10^3\text{ cm}^3}\right) = 10^3\text{ L}$. Thus,

$$rate = 7.14\times10^{-2}\frac{\text{gal}}{\text{s}}\left(\frac{3.786\text{ L}}{1\text{ gal}}\right)\left(\frac{1\text{ m}^3}{10^3\text{ L}}\right) = \boxed{2.70\times10^{-4}\text{ m}^3/\text{s}}$$

(c) $t = \dfrac{V_{\text{filled}}}{rate} = \dfrac{1.00\text{ m}^3}{2.70\times10^{-4}\text{ m}^3/\text{s}} = 3.70\times10^3\text{ s}\left(\dfrac{1\text{ h}}{3600\text{ s}}\right) = \boxed{1.03\text{ h}}$

1.75 The volume of paint used is given by $V = Ah$, where A is the area covered and h is the thickness of the layer. Thus,

$$h = \frac{V}{A} = \frac{3.79\times10^{-3}\text{ m}^3}{25.0\text{ m}^2} = 1.52\times10^{-4}\text{ m} = 152\times10^{-6}\text{ m} = \boxed{152\ \mu\text{m}}$$

1.76 **(a)** For a sphere, $A = 4\pi R^2$. In this case, the radius of the second sphere is twice that of the first, or $R_2 = 2R_1$.

Hence, $\dfrac{A_2}{A_1} = \dfrac{4\pi R_2^2}{4\pi R_1^2} = \dfrac{R_2^2}{R_1^2} = \dfrac{(2\,R_1)^2}{R_1^2} = \boxed{4}$

(b) For a sphere, the volume is $V = \dfrac{4}{3}\pi R^3$

Thus, $$\frac{V_2}{V_1} = \frac{(4/3)\pi R_2^3}{(4/3)\pi R_1^3} = \frac{R_2^3}{R_1^3} = \frac{(2R_1)^3}{R_1^3} = \boxed{8}$$

1.77 The estimate of the total distance cars are driven each year is

d = (cars in use)(distance traveled per car) = $(100 \times 10^6 \text{ cars})(10^4 \text{ mi/car})$

$= 1 \times 10^{12}$ mi

At a rate of 20 mi/gal, the fuel used per year would be

$$V_1 = \frac{d}{rate_1} = \frac{1\times10^{12}\text{ mi}}{20\text{ mi/gal}} = 5\times10^{10}\text{ gal}$$

If the rate increased to 25 mi/gal, the annual fuel consumption would be

$$V_2 = \frac{d}{rate_2} = \frac{1\times10^{12}\text{ mi}}{25\text{ mi/gal}} = 4\times10^{10}\text{ gal}$$

and the fuel savings each year would be

$$savings = V_1 - V_2 = 5\times10^{10}\text{ gal} - 4\times10^{10}\text{ gal} = \boxed{1\times10^{10}\text{ gal}}$$

1.79 **(a)** The Sun's radius is about 6.96×10^5 km and Earth's radius is about 6.38×10^3 km. The number of Earths that could fit inside the Sun, N_{ES}, is approximately equal to the ratio of the Sun's volume to Earth's volume:

$$N_{ES} = \frac{V_{\text{Sun}}}{V_{\text{Earth}}} = \frac{\frac{4}{3}\pi R_{\text{Sun}}^3}{\frac{4}{3}\pi R_{\text{Earth}}^3} \approx \boxed{1\times10^6}$$

The Sun's radius is about 10^2 times Earth's radius, so its volume (which scales as radius cubed) is about 10^6 times Earth's volume. Approximately a million Earths could fit inside the Sun.

(b) Similarly, the Moon's radius is about 1.74×10^3 km so that N_{ME}, the number of Moons that could fit inside the Earth, is

$$N_{ME} = \frac{V_{\text{Earth}}}{V_{\text{Moon}}} = \frac{\frac{4}{3}\pi R^3_{\text{Earth}}}{\frac{4}{3}\pi R^3_{\text{Moon}}} \approx \boxed{50}$$

Approximately 50 Moons could fit inside the Earth.

1.80 Assume a sneeze lasts about 3 seconds. One day equals 8.64×10^4 s which can be divided into $8.64/3 \times 10^4 = 2.88 \times 10^4$ 3-second intervals. If each of Earth's 7.3 billion people sneezes 3 times per day, then 3(7.3 billion) = 21.9 billion sneezes occur each day. The number of sneezes occurring during any particular 3-second interval is then (rounded to 1 significant figure):

$$\frac{21.9\times10^9 \text{ sneezes/day}}{2.88\times10^4 \text{ (3-sec intervals)/day}} \approx \boxed{1\times10^6 \text{ sneezes/(3-sec interval)}}$$

During any given 3-second interval, approximately one million people from around the world will sneeze.

1.81 The volume of the Milky Way galaxy is roughly

$$V_G = At = \left(\frac{\pi d^2}{4}\right)t = \frac{\pi}{4}(10^{21}\text{ m})^2(10^{19}\text{ m}) \text{ or } V_G \sim 10^{61}\,\text{m}^3$$

If, within the Milky Way galaxy, there is typically one neutron star in a spherical volume of radius $r = 3 \times 10^{18}$ m, then the galactic volume per neutron star is

$$V_1 = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi (3\times 10^{18}\ \text{m})^3 = 1\times 10^{56}\ \text{m}^3 \quad \text{or} \quad V_1 \sim 10^{56}\,\text{m}^3$$

The order of magnitude of the number of neutron stars in the Milky Way is then

$$n = \frac{V_G}{V_1} \sim \frac{10^{61}\ \text{m}^3}{10^{56}\ \text{m}^3} \quad \text{or} \quad \boxed{n \sim 10^5 \text{neutron stars}}$$